Summer Challenge (due August 31)
Fix a circle $\$ \backslash$ Gamma\$. Let $\$ T=$ |triangle $A B C \$$ be a triangle inscribed in the circle $\$ \backslash$ Gamma\$ which is not a right triangle and let $\$ H \$$ be the orthocenter of $\$ T \$$. The line \$HA\$ intersects the circle \$\Gamma\$ at \$A\$ and at a second point \$A_1\$ (which can be \$A\$ if the line is tangent to the circle). Likewise, the line \$HB\$ intersects \$ $\$$ Gamma\$ at a second point \$B_1\$, and the line \$HC\$ intersects \$|Gamma\$ at a second point \$C_1\$. The t triangle \$|triangle A_1B_1C_1\$ is again inscribed in \$\Gamma\$. We denote this triangle by । \$ $\$$ Phi $(T) \$$. Warning: $\$ 1$ Phi $(T) \$$ can be a right triangle.
I
I a) Show that triangles $\$ T \$$ and $\$ 1 \operatorname{Phi}(T) \$$ are congruent if and only if either $\$ T \$$ is equilateral or the angles of $\$ T \$$ are $\$ 1 \mathrm{pi} / 7 \$, \$ 2 \backslash \mathrm{pi} / 7 \$, \$ 4$ pi/ $7 \$$.
' b) For every integer $\$ k>0 \$$ find the number $\$ t \_k \$$ of non-congruent triangles $\$ T \$$ inscribed in $\$ \backslash$ Gamma\$ such that $\$ \mid P h i^{\wedge} k(T) \$$ and $\$ T \$$ are congruent. Here $\$ \mid P h i \wedge k \$$
denotes the composition $\$ \backslash$ Philcirc \Phi \circ \dots \circ\Phi\$ of $\$ \backslash$ Phi\$ with itself $\$ k \$$ times. Thus, according to a), we have $\$ t 1=2 \$$.
c) Is it true that if $\$ \mid \mathrm{Phi}^{\wedge} \mathrm{k}(\mathrm{T}) \$$ and $\$ \mathrm{~T} \$$ are congruent then $\$ \mid \mathrm{Phi}{ }^{\wedge} \mathrm{m}(\mathrm{T})=\mathrm{T} \$$ for some $\$ \mathrm{~m} \$$ ?
d) Find and prove your own results about $\$ 1$ Phi\$.

From:
http://www2.math.binghamton.edu/ - Binghamton University Department of Mathematics and Statistics

Permanent link:
http://www2.math.binghamton.edu/p/pow/summer_challenge


Last update: 2020/08/31 05:30

