Summer Challenge (due August 31)

Fix a circle \$\Gamma\$. Let \$T=\triangle ABC\$ be a triangle inscribed in the circle \$\Gamma\$ which is not a right triangle and let \$H\$ be the orthocenter of \$T\$. The line \$HA\$ intersects the circle \$\Gamma\$ at \$A\$ and at a second point \$A_1\$ (which can be \$A\$ if the line is tangent to the circle). Likewise, the line \$HB\$ intersects \$\Gamma\$ at a second point \$B_1\$, and the line \$HC\$ intersects \$\Gamma\$ at a second point \$C_1\$. The triangle \$\triangle A_1B_1C_1\$ is again inscribed in \$\Gamma\$. We denote this triangle by \$\Phi(T)\$. Warning: \$\Phi(T)\$ can be a right triangle.

I a) Show that triangles T and $\Phi(T)$ are congruent if and only if either T is quilateral or the angles of T are $\phi(T)$, $2\phi(T)$, $4\phi(T)$.

b) For every integer $k>0\$ find the number t_k of non-congruent triangles $T\$ inscribed in $\$ and t_k and t_k are congruent. Here θ_k is constrained in θ_k and θ_k and t_k are congruent. Here θ_k is denotes the composition θ_k is circ \Phi \circ \ldots \circ \Phi \s of θ_k is the times. Thus, according to a), we have $t_1=2$.

c) Is it true that if $\Lambda(T)$ and T are congruent then $\Lambda(T)=T$ for some m?

d) Find and prove your own results about \$\Phi\$.

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